**Convolution**

**Example-1:** The impulse response of LTI system is- **h[n] = {1, 2, 1, -1}** and the input sequence is **x[n] = {1, 2, 3, 1}**. Determine y[n].

k => xlow = 0, xhigh = 3 |

**0 ≤ k ≤ 3**

n => hlow + xlow = -1 + 0 = -1

hhigh + xhigh = 2+3 = 5 |

**-1 ≤ n ≤ 5**

**Soln:** We have -

h[n] = {1, 2, 1, -1}

x[n] = {1, 2, 3, 1}

We know

y[n]

y [-1] =

= x [0].h [-1] + x [1].h [-2] + x [2].h [-3] + x [3].h [-4]

= 1.1 + 2.0 + 3.0 + 1.0

= 1 + 0 + 0 + 0

= 1

y [0] =

= x [0].h [0] + x [1].h [-1] + x [2].h [-2] + x [3].h [-3]

= 1.2 + 2.1 + 3.0 + 1.0

= 2 + 2 + 0 + 0

= 4

y [1] =

= x [0].h [1] + x [1].h [0] + x [2].h [-1] + x [3].h [-2]

= 1.1 + 2.2 + 3.1 + 1.0

= 1 + 4 + 3 + 0

= 8

y [2] =

= x [0].h [2] + x [1].h [1] + x [2].h [0] + x [3].h [-1]

= 1.-1 + 2.1 + 3.2 + 1.1

= -1 + 2 + 6 + 1

= 8

y [3] =

= x [0].h [3] + x [1].h [2] + x [2].h [1] + x [3].h [0]

= 1.0 + 2.-1 + 3.1 + 1.2

= 0 – 2 + 3 + 2

= 3

y [4] =

= x [0].h [4] + x [1].h [3] + x [2].h [2] + x [3].h [1]

= 1.0 + 2.0 + 3.-1 + 1.1

= 0 + 0 – 3 + 1

= -2

y [5] =

= x [0].h [5] + x [1].h [4] + x [2].h [3] + x [3].h [2]

= 1.0 + 2.0 + 3.0 + 1.-1

= 0 + 0 + 0 – 1

= -1

So, y[n] = {1, 4, 8, 8, 3, -2, -1}

………………. O ……………………

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **x[n]** | **1** | **2** | **-1** | |
| **4** | 4 | 8 | -4 |
| **1** | 1 | 2 | -1 |
| **2** | 2 | 4 | -2 |
| **5** | 5 | 10 | -5 |

**Example-2:** The impulse response of LTI system is- **h[n] = {1, 2, -1}** and the input sequence is **x[n] = {4, 1, 2, 5}**. Determine y[n].

**h[n]**

Using the formula-

y[n]

We get,

y[n] = {4, 9, 0, 8, 8, -5}

**Correlation**

**Example-1:** Determine the Cross-Correlation sequence rxy (l) of the following sequences: -

x[n] = [1, 2, 3, 4] and y[n] = [4, 3, 2, 1].

xlen = 4 and ylen = 4

k => xlow = 0, xhigh = 3 | **0 ≤ n≤ 3**

tlen = xlen+ylen-1 = 4+4-1 = 7

lmin = -(tlen – xlen) = -(7 – 4) = -3

lmax = tlen – ylen = 7 – 4 = 3

**-3 ≤ l ≤ 3**

**Sol­n­:** We have,

x[n] = [1, 2, 3, 4] and

y[n] = [4, 3, 2, 1].

We know the equation for Cross-Correlation –

rxy (l)

rxy (-3)

= x [0]. y [3] + x [1]. y [4] + x [2]. y [5] + x [3]. y [6]

= 1.1 + 2.0 + 3. 0 + 4.0

= 1 + 0 + 0 + 0

= 1

rxy (-2)

= x [0]. y [2] + x [1]. y [3] + x [2]. y [4] + x [3]. y [5]

= 1.2 + 2.1 + 3. 0 + 4.0

= 2 + 2+ 0 + 0

= 4

rxy (-1)

= x [0]. y [1] + x [1]. y [2] + x [2]. y [3] + x [3]. y [4]

= 1.3 + 2.2 + 3. 1 + 4.0

= 3 + 4 + 3 + 0

= 10

rxy (0)

= x [0]. y [0] + x [1]. y [1] + x [2]. y [2] + x [3]. y [3]

= 1.4 + 2.3 + 3. 2 + 4.1

= 4 + 6 + 6 + 4

= 20

rxy (1)

= x [0]. y [-1] + x [1]. y [0] + x [2]. y [1] + x [3]. y [2]

= 1.0 + 2.4 + 3. 3 + 4.2

= 0 + 8 + 9 + 8

= 25

rxy (2)

= x [0]. y [-2] + x [1]. y [-3] + x [2]. y [-4] + x [3]. y [-5]

= 1.0 + 2.0 + 3. 4 + 4.3

= 0 + 0 + 12 + 12

= 124

rxy (3)

= x [0]. y [-3] + x [1]. y [-2] + x [2]. y [-1] + x [3]. y [0]

= 1.0 + 2.0 + 3. 0 + 4.4

= 0 + 0 + 0 + 16

= 16

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **y[n]**  **x[n]** | **1** | **2** | **3** | | **4** | |
| **1** | 1 | 2 | 3 | 4 | |
| **2** | 2 | 4 | 6 | 8 | |
| **3** | 3 | 6 | 9 | 12 | |
| **4** | 4 | 8 | 12 | 16 | |

So, we have-

rxy (l) = {1, 4, 10, 20, 25, 24, 16}

**DFT**

**Example-1:** Find the DFT of the sequence x[n] = {1, 1, 0, 0}. Sketch the magnitude and phase spectrum, real part and imaginary part. Also find the power spectrum: -

We know the equation for Cross-Correlation –

X(k)

Here, k and n = 0, 1, 3……... N-1

X (0) =

=

= x[0] + x[1] + x[2] + x[3]

= 1 + 1 + 0 + 0

= 2

X(1) =

=

= 1 – j

X (2) = 0

X (3) = 1 + j

**DFT:** X (k) = {2, 1 – j, 0, 1 + j}

**Real Part:**  Real(X(k)) = {2, 1, 0, 1}

**Imaginary part:** Imag(X(k)) = {0, -1, 0, 1}

**Magnitude Spectrum:**  abs(X(k)) = {2, }

**Phase Spectrum:**  phase(X(k)) = {0, -45, -90, 45} # in degree

**Power Spectrum:** pow(abs(X(k)), 2) = {4, 2, 0, 2}